

## **Real options: application on a viability study of a new project in the real estate sector.**

### **Abstract:**

This article deals with the usage of the real options method as a tool for aiding the economic-financial analysis of a new venture in a real state company, based on a case-study. Studies of new ventures were selected in Minas Gerais, São Paulo and Brasília, totalling 14 ventures presented in the second semester of 2006. The sample is an intentionally selected viability study of a new venture. The work sought to establish a comparison between the results obtained through traditional financial indices, resulting from an idealized Cash Flow project, and the possible results obtained from this project real options model. The research was also used to build, within the real options model, the alternative of abandoning the project, including all of the calculations and analyses which are inherent to the matter. Since this is a viability study of a new venture, there was no literature available that could help to determine the project volatility, which is an important variable for calculating the real option. The results of this work lead to the conclusion that the methodology can be applied to similar viability studies, since it brought a new perspective for decision-making. Based on what has been said, the conclusion that was reached is that the study offers arguments and information that allow for better decision-making regarding the approval or not of a new venture feasibility study. It also provides useful parameters for defining the best moment to execute the option of abandoning the project, in case this is the developer's decision.

**Key words:** Real options method; economic-financial method; abandonment option.

### **1 INTRODUCTION**

Adopting new investment-decision strategies in companies, such as the real options methodology, is a gradual process, but one of high efficiency and which may aid decision-making. A real option is the opportunity, but not the obligation, of undertaking an action at a predetermined cost and on a predetermined period, may this action be one of deferring, expanding, contracting, converting or abandoning an investment.

As regarding the usage of the real options model, it may be simpler when the projects and companies present well defined options before market uncertainties of price behaviour, demand, entrance of new competitors, knowledge of new businesses, research and development investments, and participation in auctions for concessions.

By adopting the mentioned methodology, one expects to answer, at the moment of the decision and even at certain periods after it, what is the value of the project and what is the value of the option that may be chosen during the venture's operational cycle.

This is the scenario in which this study is inserted. An approach focused on the resources offered in order to generate value for the stockholder becomes feasible by contemplating the manager's role and by incorporating the uncertainty that exists in appraisal models. The process of appraising companies begins by understanding that generating value is one of its main objectives. Creating value for the stockholder is a central topic in modern finance.

Bearing in mind the research's focus, the option adopted was of abandoning the project, which, after being determined, may be added to the traditional result indices, especially the Net Present Value (NPV). The study also shows that, by using the real options method, when considering the project's total value one must take into consideration the value

of the uncertainties and flexibilities, which are important for the company to better identify its project's risks.

In such way, the broker must decide whether he will invest or not in a certain project that will provide a certain value as a result. By using real options, in this case the abandonment option, the broker will be able to quantify the value of the project being analysed, knowing that he will eventually have the option of abandoning it in case he wishes to do so.

One of the possible contributions this work may bring seems to lie in the critical reflexion that was attempted concerning the benefits of the practical application of real options models whose methodology proposal is to provide appraisal techniques for investment projects considering the value of uncertainty and flexibility.

The choice for the real abandonment option is justified by the fact that it shows the necessity of knowing other investment-decision parameters in real estate projects.

In addition to this, we investigated how to reduce the rejection possibilities of projects that, even though having been disapproved for not reaching some of the indices pre-established by the company for approving its projects, could generate satisfactory results if they were appraised by a viability study which included an option that could improve its value, such as abandonment.

It is worth mentioning that a study on the real estate sector was developed in the city of Rio de Janeiro, in 2001, called "Application of real options on the residential real estate market" ("*Aplicação de opções reais no mercado imobiliário residencial*"). It focused on Rio de Janeiro, yet in a more market-oriented approach, applying a different model from the one used in this work.

The Brazilian real estate sector is represented here by one of the main companies of the sector, with a total of 14 proposals in Minas Gerais, São Paulo and Brasília.

The developer incorporates, builds and sells high-end and extremely high-end residential and commercial developments in the South-eastern region of Brazil. It also acts as contractor of civilian and industrial constructions for the private enterprise and the public organizations. It also possessed, in September 2006, approximately two thousand workers, 90% of whom were assigned to constructions. The cash administration is centralised.

The sample is an intentionally selected new venture, which was considered unviable by the company's managers according to the premises internally adopted for the approval of the projects. The chosen project was selected among 14 proposals presented to *Construtora Beta Ltda* during the second semester of 2006.

It was also essential to use a viability study for the proposal of a new enterprise, based on the CF originally foreseen on the moment of deciding whether or not to approve the viability study. In this case, the choice was made in favour of a rejected proposal, mainly due to the fact that it presented a negative NPV. The value of the FC's option was then calculated and added to the original NPV. The result was a new NPV that could make this project appraised again or not, taking into consideration the option value.

It is worth mentioning that the venture offered enough information in order to assess, in the real options method, means which allow its usage as an economic-financial analysis tool.

To collect the data that is essential for the research, a comparison was made between the results obtained through traditional financial indices, resulting from an idealised CF from a certain project, and the possible results obtained through this project's real options. We took into consideration, in this case, the uncertainties and flexibilities during that time.

During the documental collection, the foreseen flow of the venture, also called original flow, was also obtained. Based on this flow a new one was developed using the volatility

calculation simulations, so as to make it possible to build the NPV tree and to calculate the options' value.

The research was also used to build, within the real options model, the option of abandoning the project, with all of the calculation and analysis systematic inherent to the matter.

The data was gathered by the researcher from January to March 2007, in the developer's office.

The analysis of the data involved many procedures: codification of the answers, tabulation of the data, and statistical calculations (GIL, 2002, p.125). In this study the tabulation of the data was made through electronic spreadsheets and by the software @Risk<sup>®</sup>, used for treating and calculating the relevant values.

The quantitative analysis technique was used as a methodological tool for obtaining a better understanding of the transformations in the viability study for the implementation of the project.

The data analysis also began from two basic premises. The first premise refers to the original DCF, which is the enterprise's viability study provided by the developer. The study was developed by the appropriate sectors within the company, and then approved by the decision-making committee. It was made in accordance to the basic premises proposed by the company, predominantly market studies. Emphasis was put on the how fast sales could be made, on the prices charged, and on the budget schedule, specifically on the prices charged and deadlines.

The second premise refers to a new flow that was created with basis on the original DCF by simulating different scenarios for the evolution of the apartment prices. In order to simulate the scenarios – pessimist, expected, and optimist – it was considered that the apartment units would be sold for cash, taking care not to compromise the revenue of the period, which is informed in the original DCF.

Seeking to establish a parallel between the variables that make up the prices evolution in the three fields, the software @Risk<sup>®</sup> was used, and it made it possible, through the "Monte Carlo Simulation", to calculate the volatility value, which is a variable necessary to calculate the real option value.

## **2 APPRAISAL AND APPLICABILITY OF THE REAL OPTIONS METHOD**

This research is essentially based on the usage of the real options method as an aiding tool for the economic-financial analysis. The object of this study is a company of the real estate sector.

Such analysis was based on the real options theory, through the binomial model, using scenarios of a CF variable for volatility calculation. To conclude the study, a comparison was made between the traditional NPV method and the real options method, identifying its convergent and divergent points. Finally, the optimum moment for abandoning a venture was investigated.

### **2.1 Details of the data gathering**

#### **2.1.1 Cash Flow**

The information necessary to make considerations about the viability study of the new venture of *Construtora Beta Ltda* was obtained through the data that was gathered, analysed and backed by specialised bibliography. It was displayed in the format of a CF projected every semester, from 2007 to 2011, as shown in Table 1

Table 1 – Cash flow: consolidated simulation

CASH FLOW	Incorporation	1		2		3		4		5		Total	Média/ semestral	
		2007_1	2007_2	2008_1	2008_2	2009_1	2009_2	2010_1	2010_2	2011_1	2011_2			
Apartments sold		3	4	4	4	6	3	15	1	-	-	40	4	
Price- calculated with volatility		410.000	446.388	486.061	529.323	576.503	627.961	684.093	745.331	0	0	585.668	450.566	
<b>Gross revenue</b>		<b>1.230.000</b>	<b>1.785.550</b>	<b>1.944.245</b>	<b>2.117.292</b>	<b>3.459.016</b>	<b>1.883.884</b>	<b>10.261.402</b>	<b>745.331</b>	<b>0</b>	<b>0</b>	<b>23.426.719</b>	<b>2.342.672</b>	
Direct taxes		(48.059)	(100.547)	(74.256)	(88.517)	(80.978)	(72.183)	(366.113)	(10.299)	0	0	(840.951)	(84.095)	
<b>Net income</b>		<b>1.181.941</b>	<b>1.685.003</b>	<b>1.869.989</b>	<b>2.028.775</b>	<b>3.378.038</b>	<b>1.811.701</b>	<b>9.895.289</b>	<b>735.031</b>	<b>0</b>	<b>0</b>	<b>22.585.768</b>	<b>2.258.577</b>	
Land + clean/maintenance/security		(403.531)	(9.954)	(16.167)	(27.139)	(30.440)	(23.532)	0	0	0	0	(510.764)	(51.076)	
Product definition + project viability		(139.477)	(121.212)	0	0	0	0	0	0	0	0	(260.689)	(26.069)	
Project cost		(316.436)	(1.244.209)	(2.020.922)	(3.392.333)	(3.804.981)	(2.941.538)	0	0	0	0	(13.720.419)	(1.372.042)	
Incorporation		(40.000)	0	0	0	0	(97.205)	0	0	0	0	(137.205)	(13.720)	
Publicity and Promotion		(529.227)	(267.897)	(174.016)	(105.419)	(19.247)	(71.002)	0	0	0	0	(1.166.809)	(116.681)	
Re-execution of the services		0	0	0	0	0	0	(68.602)	(68.602)	(68.602)	(68.602)	(274.409)	(27.441)	
Project Administration		(51.649)	(154.631)	(202.362)	(319.889)	(348.770)	(274.875)	(200.610)	(5.643)	0	0	(1.558.429)	(155.843)	(17.628.723)
<b>EBITDA</b>		<b>(298.379)</b>	<b>(112.900)</b>	<b>(543.479)</b>	<b>(1.816.004)</b>	<b>(825.400)</b>	<b>(1.596.451)</b>	<b>9.626.077</b>	<b>660.786</b>	<b>(68.602)</b>	<b>(68.602)</b>	<b>4.957.045</b>	495.705	<b>22%</b> s/ RL
Amortization														
Depreciation														
EBIT		(298.379)	(112.900)	(543.479)	(1.816.004)	(825.400)	(1.596.451)	9.626.077	660.786	(68.602)	(68.602)	4.957.045	495.705	
Loan interest		0	0	0	(66.349)	(191.791)	(321.340)	(273.918)	0	0	0	(853.399)	(85.340)	
Additional Interest (Exposure of CF)														
IR/CSS														
<b>Operating Cash Flow</b>		<b>(298.379)</b>	<b>(112.900)</b>	<b>(543.479)</b>	<b>(1.882.353)</b>	<b>(1.017.191)</b>	<b>(1.917.791)</b>	<b>9.352.159</b>	<b>660.786</b>	<b>(68.602)</b>	<b>(68.602)</b>	<b>4.103.647</b>	<b>410.365</b>	
Capex (Invest + NCG)	(108.917)													
Var. NCG														
<b>Stockholders cash flow</b>	<b>(108.917)</b>	<b>(298.379)</b>	<b>(112.900)</b>	<b>(543.479)</b>	<b>(1.882.353)</b>	<b>(1.017.191)</b>	<b>(1.917.791)</b>	<b>9.352.159</b>	<b>660.786</b>	<b>(68.602)</b>	<b>(68.602)</b>	<b>4.103.647</b>	<b>410.365</b>	
Residual Value = stock holder basis														
<b>Final result - stockholders</b>	<b>(108.917)</b>	<b>(298.379)</b>	<b>(112.900)</b>	<b>(543.479)</b>	<b>(1.882.353)</b>	<b>(1.017.191)</b>	<b>(1.917.791)</b>	<b>9.352.159</b>	<b>660.786</b>	<b>(68.602)</b>	<b>(68.602)</b>	<b>4.103.647</b>	<b>363.157</b>	

It is worth mentioning that, despite the fact that the CF presents a positive NPV, the project would not be approved by the company, since the NPV is below the value which the company considered as a premise for such venture. Since the company had not, until that moment, made an option to abandon its ventures, this option was made respecting the existing contract clauses. The option value was also analysed and how it could add value to the project, making it viable when the new NPV result was obtained.

### 2.1.2 Simulation of scenarios for volatility calculation

Based on the gathered data it was also relevant to analyse the scenarios for volatility calculation, since there was no historical basis to consult as this was a viability study for a new enterprise, different of those usually developed by the company. The calculation parameters took into consideration the simulation of results in new ventures' viability studies. Three scenarios were projected: the most expected one plus two others, called optimist and pessimist, for the account "Price – calculated with volatility". It is important to note that this variable is the most important one for the study of volatility, using variation percentages for the prices informed and charged by *Construtora Beta Ltda.*

In this case, the developer used, for the expected scenario, the projected variation for the period given by the *Índice Nacional da Construção Civil – INCC* (National Real Estate Index). For the pessimist scenario this same index was used with a reduction of 17.52%, following the company's criteria. For the optimist scenario the variation projected for the Certificado de Depósito Interbancário – CDI (*Interbank deposit certificate*), which is tied to the Selic index, Brazil's basic interest rate, was used. This rate is used as a parameter for the payoffs offered by the banking institutions in their traditional funds applications and in their Bank Deposit Certificates (BDC). Table 2 displays the variations projected for the studied venture.

Table 2 – Variations projected for the venture

VARIATION	PERCENTAGE
PESSIMIST	65,90%
EXPECTED	79,89%
OPTIMIST	99,57%

Note: Variation of the unit price, from the first to the last price charged.

Considering these aspects, and taking into account that the viability study model does not have a historical and official basis for its volatility, the volatility of the asset-object was projected. The focus was on the volatility of the main account of the CF referring to the viability study of the new venture, in this case, the account "Price – calculated with volatility".

The volatility estimation of the subjacent-asset subjected to risk (asset-object) is in accord to Copeland and Antikarov (2001, p. 245), who claim, based on the Monte Carlo approach, that the existence of estimates for the stochastic properties of the variables that drive the volatility is called *critical accounts*. In this case, the account that was used is "Average unit price" (Table 3).

Table 3 – Economic premises used

YEAR	2007_1	2007_2	2008_1	2008_2	2009_1	2009_2
Expected Quantity	3	4	4	4	6	3
Pessimist value	410.000	440.750	473.806	509.342	547.542	588.608
Expected value	410.000	445.875	484.889	527.317	573.457	623.635
Optimist quantity	410.000	452.538	499.488	551.310	608.509	671.641
Average price - volatility impact	410.000	446.388	486.061	529.323	576.503	627.961
<b>GROSS REVENUE</b>	<b>1.230.000</b>	<b>1.785.550</b>	<b>1.944.245</b>	<b>2.117.292</b>	<b>3.459.016</b>	<b>1.883.884</b>

YEAR	2010_1	2010_2	2011_1	2011_2	TOTAL/ Var
Expected Quantity	15	1			40
Pessimist value	632.754	680.210			65,90%
Expected value	678.203	737.545			79,89%
Optimist quantity	741.324	818.237			99,57%
Average price - volatility impact	684.093	745.331			
<b>GROSS REVENUE</b>	<b>10.261.402</b>	<b>745.331</b>			<b>23.426.719</b>

### 2.1.3 Discount rate

The discount rate is adjusted to the risk of the project. The methodology most used to assess the discount rate is the Weighted Average Cost of Capital (WACC). It is necessary to estimate the intended capital structure, the cost of third party capital and the cost of your own capital (MINARDI, 2004).

Relevant analyses were made about the discount rates used in the projected CF. The WACC was used to find the static NPV (original NPV) of the project (COPELAND; ANTIKAROV, 2001).

It is important to notice that an inflation of 4% was assumed in order to calculate the discount rate. This is a more conservative estimation than the inflation of 3,9% projected for 2007 measured by the Extended Consumer Price Index (IPCA - *Índice de preços ao consumidor amplo*), which was given by the *Banco Central do Brasil* – BACEN (Central Bank of Brazil) – in its trimester report, released on December 2006 (Table 4).

Table 4 – Discount rate

WEIGHTED AVERAGE CAPITAL COST						
WACC	Structure	Cost	Net capital	Weighting	Capital	
Third parties' capital	100,00%	22,42%	14,7970%	14,80%	0	Kt
Own capital	0,00%	0,00%	0,00%	0,00%	0	Ke
<b>WACC</b>				<b>14,80%</b>	<b>0</b>	
<b>WACC + INFLATION</b>				<b>18,80%</b>	<b>0</b>	<b>YEARLY</b>

## 2.2 Monte Carlo simulation analysis

### 2.2.1 Volatility and probable NPV calculation

As regarding the CF of the new enterprise study, the asset-object to be used for calculating the real options, whose possible uncertainties and flexibilities may decisively influence the final result of the project, was defined. It is essential to say that the chosen asset-object was the NPV of the project, since it is the main variable for deciding the viability of projects in the Beta company.

A total of 10.000 interactions were made for each one of the three simulations, as it was demonstrated in the sub-sections: a summary of the data; a distribution curve; an accumulated distribution curve of the NPV; a summary of the statistical parameters; a sensibility analysis; and the sensibility analysis of the NPV through linear regression.

The obtained results are described in the following sections which received the name of the table produced as a result.

### 2.2.1.1 Data summary

The information is usually presented in tables. In the memory of the information used to make the simulations, is stated the number of simulations made (3) and the number of interactions in each (10.000). There are 8 previously listed stochastic input variables (inputs). Another important point is the fact that these variables are relevant for the CF model of the studied enterprise (Table 5).

Table 5 – Data summary

Summary Information	
Workbook Name	FCD Construturo II.xls
Number of Simulations	3
Number of Iterations	10000
Number of Inputs	8
Number of Outputs	1
Sampling Type	Monte Carlo
Simulation Start Time	4/4/2007 21:27
Simulation Stop Time	4/4/2007 21:29
Simulation Duration	00:01:26
Random Seed	1

### 2.2.1.2 Distribution curve and NPV accumulated distribution curve

In the correlation of the NPV distribution and the most probable distribution, one may verify that the result of the NPV reveals that there is a 90% probability of the NPV being in the interval between R\$1,044,300.00 and R\$1,688,600.00 (Figure 1). It can be said that in the results one may also find the most probable NPV to be realised by the enterprise, at the value of R\$1,352,366.00.

Observing the distribution made by the NPV, it can be seen by the simulations made that the accumulated NPV curve has a different shape. In the curve one may directly read the occurrence probability (ordinates) of a certain NPV value (abscissas), as shown in Figure 2.

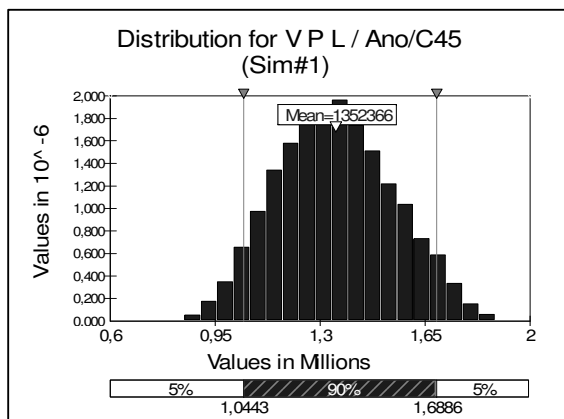


Figure 1 – Distribution curves

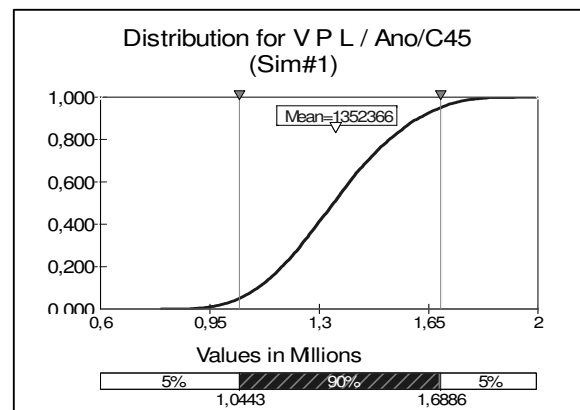


Figure 2 – NPV accumulated distribution curve

### 2.2.1.4 Statistic parameters summary

Aspects related to the statistical summary are also presented in tables, focusing on the main statistics obtained through post-simulation. It was observed that the results that most interfered in the study were the average NPV value (mean) and the standard deviation value (std dev), since they are the two variables used to calculate the volatility, which was obtained through Formula 1.

Volatility = Std Dev / Mean. Therefore, the result found is 14.35% (1)

Table 6 – Statistic parameters summary

Statistic	Value	%tile	Value
Minimum	793.924	5%	1.044.302
Maximum	1.995.106	10%	1.101.846
<b>Mean</b>	<b>1.352.366</b>	<b>15%</b>	<b>1.145.170</b>
<b>Std Dev</b>	<b>194.085</b>	<b>20%</b>	<b>1.179.806</b>
Variance	37669071387	25%	1.212.028
Skewness	0,139166994	30%	1.240.771
Kurtosis	2,554505971	35%	1.266.887
Median	1.345.703	40%	1.293.084
Mode	1.354.164	45%	1.319.976
Left X	1.044.302	50%	1.345.703
Left P	5%	55%	1.370.924
Right X	1.688.567	60%	1.396.506
Right P	95%	65%	1.423.888
Diff X	644.266	70%	1.454.971
Diff P	90%	75%	1.487.137
#Errors	0	80%	1.523.657
Filter Min		85%	1.566.521
Filter Max		90%	1.616.724
#Filtered	0	95%	1.688.567

### 2.2.1.5 Sensibility analysis

The sensibility of the accounts in relation to the NPV may be represented in the format of a table or graphically. In this case, the regression of the accounts in relation to the NPV over time is represented. The sensibility analysis may be seen in Table 7.

Table 7 – Sensibility analysis

Rank	Sensitivity		
	Name	Regr	Corr
#1	Average Price - Volatility impact / 20	0,936	0,938
#2	Average Price - Volatility impact / 20	0,251	0,240
#3	Average Price - Volatility impact / 20	0,158	0,155
#4	Average Price - Volatility impact / 20	0,125	0,107
#5	Average Price - Volatility impact / 20	0,084	0,100
#6	Average Price - Volatility impact / 20	0,073	0,088
#7	Average Price - Volatility impact / 20	0,042	0,032
#8	Average Price - Volatility impact / 20	0,000	0,000
#9			

### 2.2.1.6 Analysis of the NPV sensibility through linear regression

As regarding the statistical regression through NPV sensibility, one may say that it is also represented in the format of a bar chart. In order to do so, the values that were created in



the formulation of the unit price scenarios in the CF became the variables used in the simulation of the volatility calculation, as shown in Figure 3.

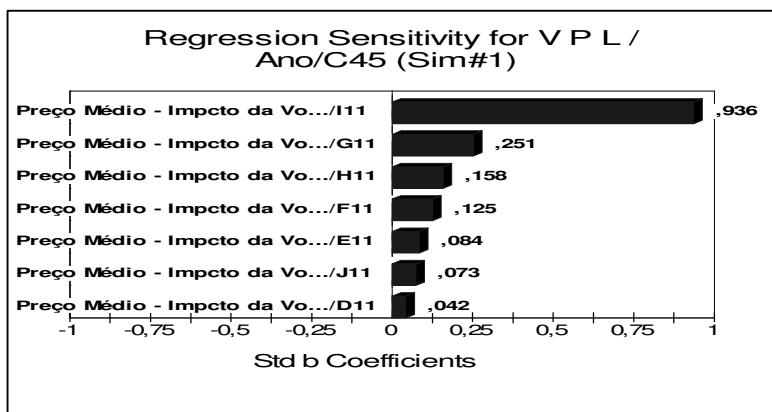


Figure 3 – NPV sensibility analysis through linear regression

## 2.3 Methods for calculating the option value

The composition of the methods for calculating the option value comprehended the following criteria: input parameters; calculated parameter; NPV tree identification; buying option's price; buying option value; selling option's price.

### 2.3.1 Volatility

Greater emphasis was put on the volatility value, since it is an important variable to the whole process. This would make it possible to seek, from the binomial model, the results that would meet the aims of this study. The calculations were made through the input of the parameters and their calculation, in which were identified all the variables that would be necessary to determine the options' values.

Bearing in mind that the chosen model is the binomial model, it was decided to correlate it with the ideas of Cox, Ross and Rubinstein (1979). According to the authors, the analysis model is the binomial one, used for pricing the options that do not pay dividends. This model is the most suitable one when the calculation of the option is made over one variable, in this case the NPV of the project.

### 2.3.2 Input parameters and calculated parameters<sup>1</sup>

As regarding the input parameters (Table 8), it is important to notice that the steps for their calculation focused on five aspects:

- effective discount rate for binomial tree – 6% yearly, the discount rate used was the risk-free rate, which is net of taxes. The 6% percentage was considered the minimum return on savings account in Brazil, which is the minimum return for national investments and is considered realistic within the scenario of the project. The calculation was based on the option value, inserted in the context of the risk-neutral appraisal. Finally, the rate was transformed in an effective semester risk-free rate. In another perspective, Assaf Neto (2005) says that this rate can be calculated using the average interest rates of the public bonds issued by the US government;
- current value of the asset-object "S" – refers to the current value of new venture of *Construtora Beta Ltda*, at present in "0". It can be said that it is the project's most probable NPV;

- c) price of execution “X” – this value is considered as the one proposed for executing the option. Since it is an abandonment option, without a price proposed for the sale of the project, it is considered, in this case, to have the same value as the asset-object (BRASIL, 2002);
- d) life option by period – comprehends the viability study of the new venture, appraised with a five year duration, distributed every semester in the studied CF, at the proprietor’s or investor’s discretion;
- e) volatility – refers to the NPV volatility calculation method. One must notice that this calculation is used to project the ascending and descending movement of the NPV over time.

The details of the input parameters definitions for the calculation of the real options were made based on the parameters that will make possible to build the format of the tree that is contained in the binomial model (Table 9).<sup>1</sup>

Table 8 – Input parameters

Input parameters	
1. Yearly risk-free rate, $r_0$	2,96%
2. Current value of the asset-object, S	1,353
3. Execution price, X	1,353
4. Option's life, in years, T	10
5. Current standard-deviation, s	14,3515%
6. Interval between each step, Dt, in years	1

Table 9 – Calculated parameters

Calculated parameters	
1. Ascending movement by step	1,15
2. Descending movement by step	0,87
3. Risk-free factor, r	1,030
4. Risk-neutral prob. (ascending)	0,567
5. Risk-neutral prob. (descending)	0,433

### 2.3.4 Identifying the NPV tree

Attention is drawn to the fact that, after defining the initial parameters, the construction of the NPV tree starts. It is based on the average value of the probable NPVs found in the simulation made using the software @Risk<sup>®</sup>. It is worth noticing that this value is referred to, in this case, asset-object “S” (Figure 4).

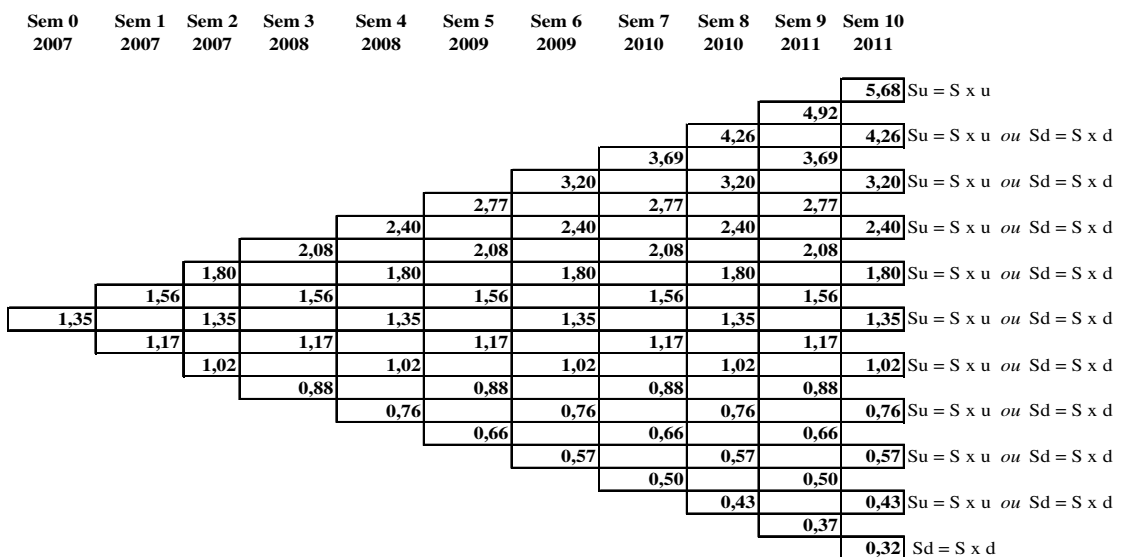


Figure 4 – Binomial tree of the asset-object’s price  
 Note: R\$ Millions.

Another point which deserves attention is that, based on this NPV, new values are calculated over time, based on the ascending and descending movement, up to the last period of the CF. In order to do so, the new values will be originated in the probabilities of the prices rising or falling, calculated based on the volatility of the project.

After building the NPV tree, the construction of the real options tree is sought, demonstrating the value of the options according to the moment that their respective NPV, shown at the NPV tree, occurs.

As pertaining to the methodology, the construction of the options tree begins from the last period going on to the preceding one. The values are successively decapitalised at the risk-free rate “ $r_0$ ”, until the option value is calculated in the first period (year zero).

One last aspect to be mentioned is the generation of two trees – one of buying options and another one of selling options, i.e., one from the viewpoint of whom might acquire the project and one from the viewpoint of whom might sell the project. For this study, the most adequate is the selling option, since it means, even if not explicitly, an abandonment option. The expression “not explicitly”, in this study, becomes important, for abandonment option does not necessarily mean that there will be a sale, even though the selling option necessarily means an abandonment option. The terminology’s use is thereby justified, the selling options tree being the one adopted for this study.

### 2.3.5 Calculating the price of the buying option

Minardi (2004), when referring to the buying option’s price, makes it clear that the acquisition of a European option “c” or an American option “C” gives the right to purchase one unit of the asset-object at the execution price “X”. But the option can never have a value greater than that of the asset “S”. Therefore, “ $c \leq S$ ” and “ $C \leq S$ ”.

According to this author’s reference, it was verified that the longer the duration of the project, the larger the volatility and the smaller the present value of the exercise. This result shows that there is a tendency of the buying value increasing as the maturity date increases, though acting in the opposite direction of a selling option value. Therefore, the larger is the remaining time of the exercise, the greater is the tendency for the buying value to increase, albeit it is not being possible to detect any aspect about its effect on the selling option.

The binomial tree for the American buying option tree can be visualised in Figure 5.

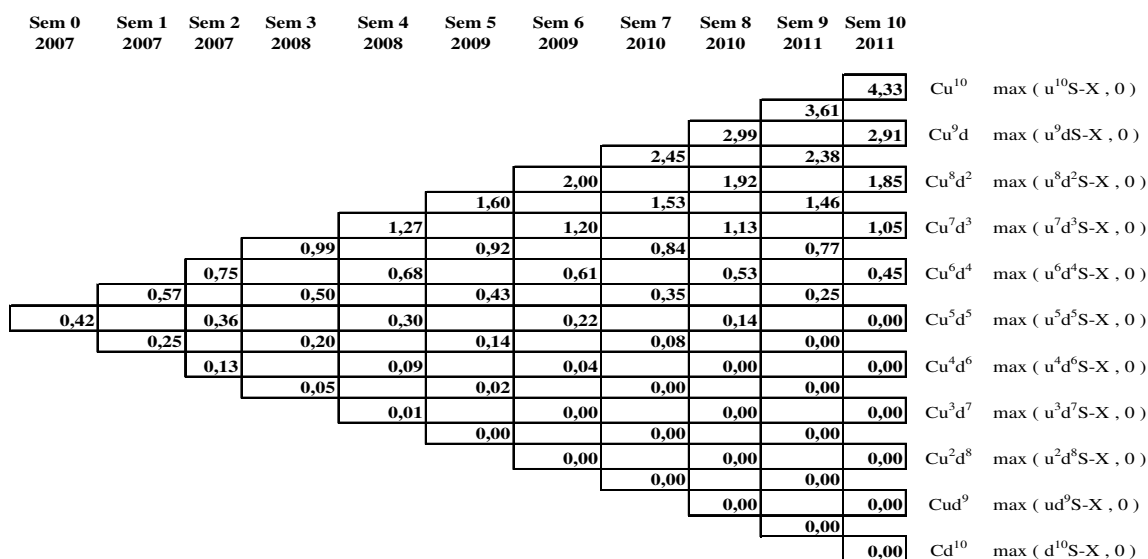


Figure 5 – American buying option Binomial tree  
Note: R\$ Millions.

### 2.3.6 Buying option value

After calculating the buying option value, demonstrated in Figure 10, it becomes essential to add to it the value of the NPV found in the NPV tree in the same period. Aspects related to this new tree demonstrate that the NPV values impacted by the buying option value in the same period must be used by the investors as an instrument for making decisions whether to invest or not in the venture.

### 2.3.7 Identifying the selling option's price

Aspects regarding the upper limit are reinforced by Minard (2004). According to the author, the European selling option “p” or the American “P” gives the right to sell one unit of the asset-object for the execution price “X”. But, independently of the asset low price, the option will never have a value greater than “X”. Therefore, “ $p \leq X$ ” and “ $P \leq X$ ”.

In such way, it may occur that the selling option is in its best moment for execution in the last period, what could mean that the asset execution has a higher value.

After the selling options tree is built, in this study called abandonment options tree, the value of the option to be added to the NPV of this project emerges, lining up in this way the result of the new value for this NPV. This value shall be the one used for the viability study of the venture being studied.

In such way, Brasil (2002, p.175) mentions that “the abandonment option value is added to the respective value of the NPV at the same moment” (Formula 2).

Abandonment option = business value without abandonment option (+) abandonment option value (2)

It is also emphasised that the selling options tree could enable the determination of the best moment to execute the abandonment option. One may infer that every time the sum of the option value and the NPV value in the NPV tree is inferior to the value of the NPV of the project, the project should be abandoned. However, if it is superior to the NPV of the project, the project should continue.

### 2.3.8 Value of the option to sell

Once the value of the option to sell has been identified (Figure 6), the NPV found in the same period NPV tree is added to it. From there on, in the new tree it is located the NPVs impacted by the value of the option to sell in the same period. The managers will thereby be able to use the tool to make the decision of abandoning or not the business. Additionally, this tool also helps to define the adequate moment to make this decision (Figure 8).

value as the project is carried out, i.e., the value of the NPV, or even foresee, over time, the best moment of abandoning it or not. In this study abandoning the project can mean the immediate sale of the asset at the current value, which will be the expected NPV plus the value of the asset selling option, compared to the execution value, or simply abandoning the project without necessarily selling it.

In this case, the real-estate developer may have a contractual deal in which it has a given period of time to carry out the project, and, if it does not receive the foreseen satisfactory results, can abandon it, giving back the acquired land and assuming the possible financial losses previously contracted.

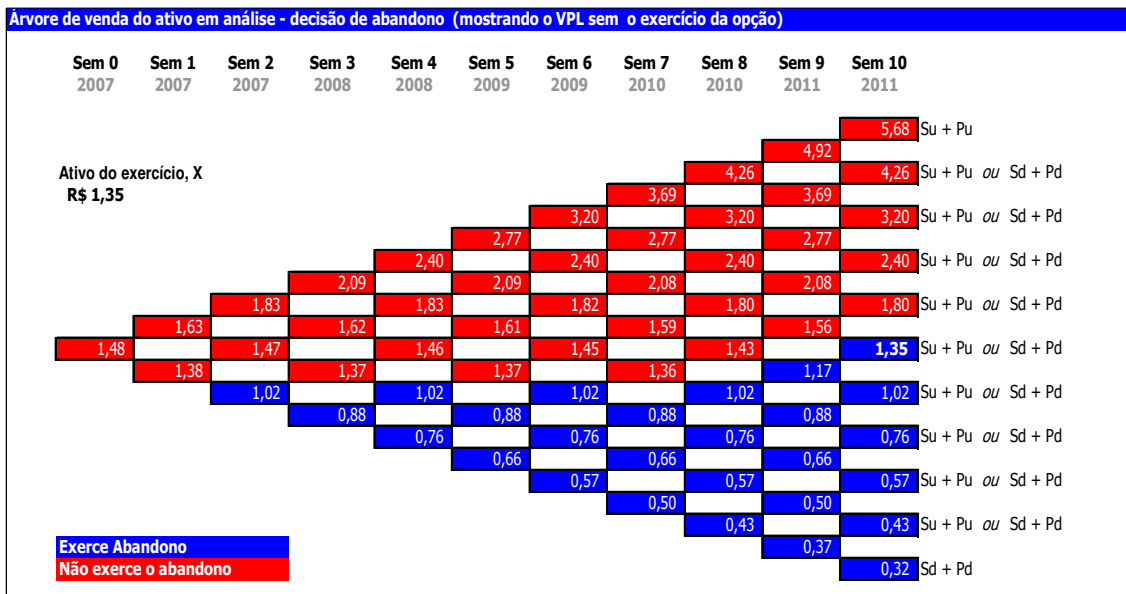


Figure 6 – Selling tree of the analysed asset – abandonment decision: showing the NPV without the execution of the option  
 Note: R\$ Millions.

Another important point is that this tree can allow the managers to check the business

It is worth mentioning that in another tree the values of the losses of not executing the option can be identified (Figure 7). In this case, in the “nodes” in which the abandonment option is identified one can see the additional value that the stockholder may not get for not executing the adequate abandonment option.

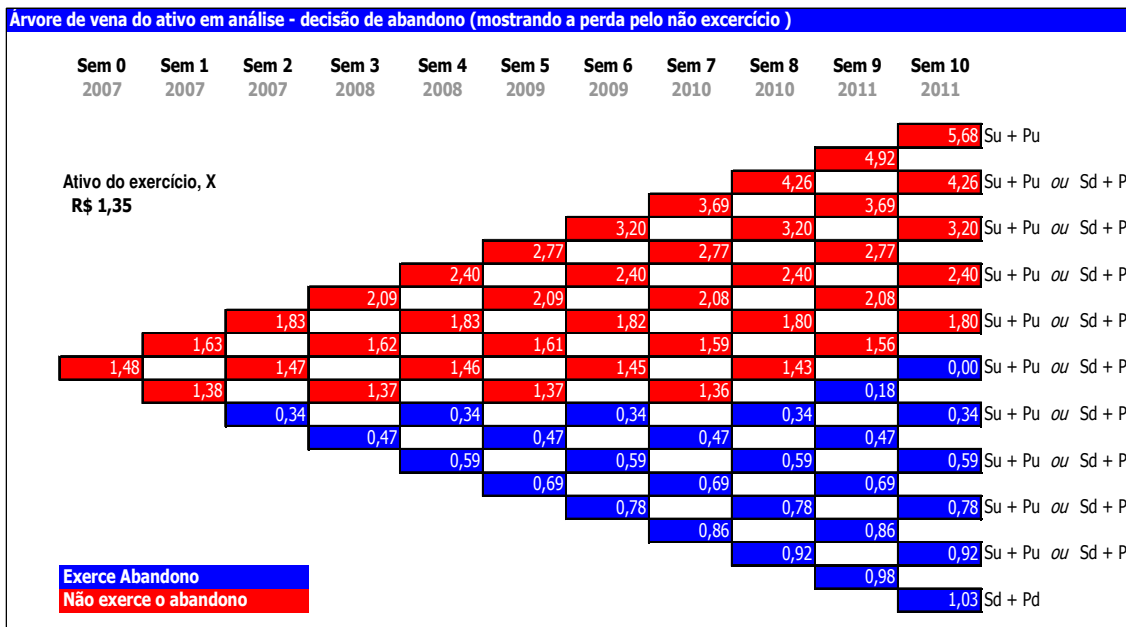


Figure 7 – Selling tree of the analysed asset – abandonment decision: showing the loss for not executing the option  
 Note: R\$ Millions.

The methodology that was adopted in this study and presented by some authors, among them Silva (2004), shows in each of the “nodes” of the tree the NPV found in the NPV

tree and, and next to it, the value of the selling option. To fulfil the objective proposed in this study, this presentation form was altered, separating all the trees.

## 2.4 Verification of the results

The appearance of uncertainties and flexibilities along the project becomes an important aspect in the results, for the methodology of real options recommends a periodic inspection of the NPV, not limiting oneself to verifying the CF components, as has been demonstrated by Copeland and Antikarov (2001).

When they deal with this matter, the authors mentioned that the project is subjected to relevant events during the period of its realisation. In the moment when an important fact occurs and alters the NPV in a certain period, it is possible that there also occurs a change in the decision of abandoning the project or not. The influence could be caused, for example, by changes in the economy or by a new competitor entering the real estate market, among other aspects.

The NPV validation model proposed in this study can be seen in Figure 8.

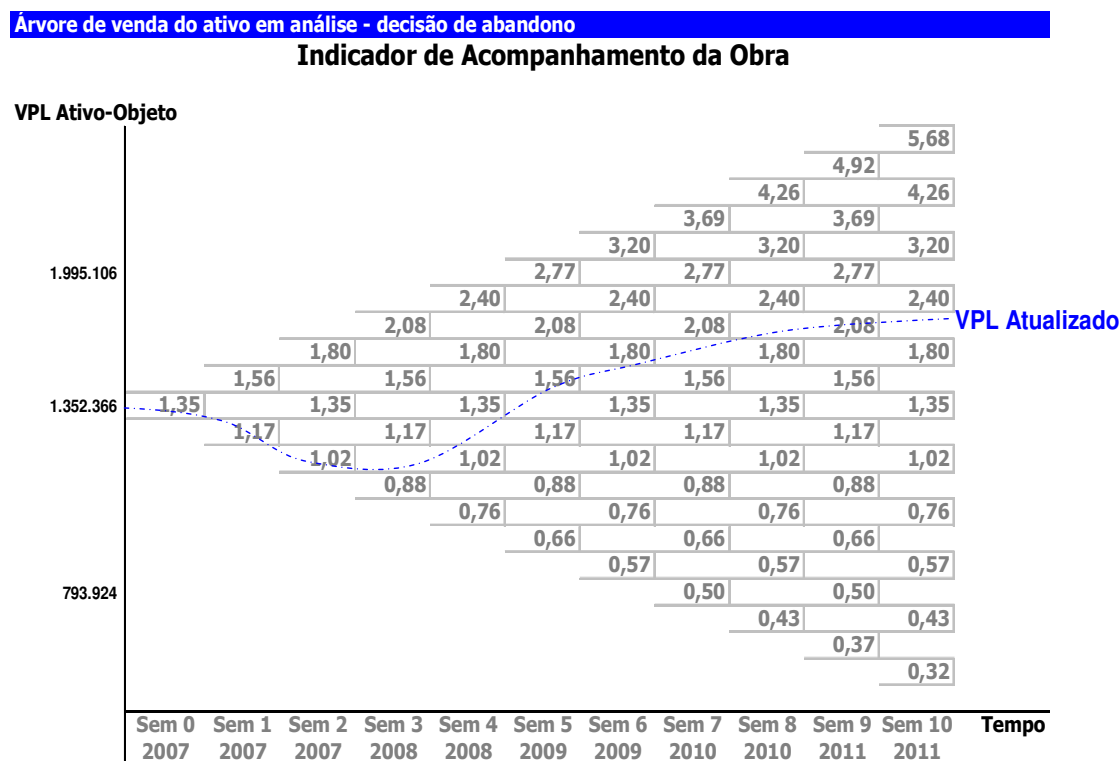


Figure 8 – Selling tree of the analysed asset – abandonment decision  
Note: R\$ Millions

## 3 CONCLUSION

This study defined an abandonment option. It was sought, therefore, to calculate the option value, adding it to the NPV of the original viability study, which was below what the company considered ideal for the new analysed project. The objective was to identify a new NPV that would contribute to the process of making the decision of whether to approve the project or not. This was done bearing in mind that, according to the methodology, one would be taking into consideration inherent factors that could possibly occur during the CF, a fact that the traditional indices cannot measure.

It is worth mentioning that, even though the CF of the viability study demonstrates a positive NPV, according to the literature and market practices, what would make an approved project (viable), was rejected by the developer, since it did not fit into the strategic parameters practiced by the company.

It was verified that the methodology adopted in this research can be used in the identification of the factors that are preponderant in the uncertainties and flexibilities that may occur along the life of a certain project and that are not considered, or yet, do not have their values calculated, in the most traditional appraisal indices.

The use of the methodology may even benefit the real-estate developer, in the sense that it will arouse, in its managers and analysts, the concern with the simulation of future scenarios that may occur along the project.

The study showed that it is possible to determine the optimum moment for abandoning a venture, according to the results presented, and it may even become an important tool for checking the CF of this venture. It emphasises the NPV, as it suggests alternatives and offers periodic values for the managers' knowledge, analysis and decision-making process. They may acquire an instrument that allows them to periodically check the project result– not only the evolution of the accounts that make up the CF, as normally happens, but also the value of the venture, and even more, the time to analyze and decide when to abandon the project or not.

Finally, the conclusion reached is that the real options method as a tool for aiding the economic-financial analysis of a real estate developer presented satisfactory results. It can, therefore, provide arguments and information that allow better decision-making regarding the approval or not of a viability study for a new project, possessing parameters useful for defining the best moment to execute the option of abandoning the project.

As a limitation of this study, attention is drawn to the point that the literature does not contain deep studies about real options for the Brazilian real estate sector, little contributing to crucial issues. One may mention, as an example, the gathering of historical bases taken from market researches done by the researcher company itself, as well as information given by entities and specific organizations of the real estate sector. It would also have been important to have obtained scientific-literary information.

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<sup>1</sup> (1) It is the annual rate of 6% used to compose the minimum remuneration of the Brazilian savings account, being used in the market as a risk-free rate. (2) Current value of the project (NPV). (3) Execution value of the project. Considered to be equal to the current value "S" (value of the asset-object), since there is not a defined proposal of buying the project. (4) It is the same as the volatility of the asset. Since it is a new project, therefore not possessing a historical basis, the NPV's volatility (standard deviation) was used, this value being obtained by using the Monte Carlo simulation and the @Risk tool.